

The University of Chicago

ON THE VELOCITY OF SOUND IN AIR
BY A NEW METHOD

A DISSERTATION
SUBMITTED TO THE FACULTY
OF THE OGDEN GRADUATE SCHOOL OF SCIENCE
IN CANDIDACY FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

DEPARTMENT OF PHYSICS

BY
KARL KELCHNER DARROW

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THE VELOCITY OF SOUND IN AIR BY A NEW METHOD

The paper following is submitted as a report on a new method for determining the velocity of sound in gases, devised especially for use with gases other than air, but apparently capable of furnishing results comparable in accuracy with those of other methods, including those applicable only to air.

In all careful investigations with gases other than air, it is necessary to inclose the gas in an air-tight chamber, generally a tube. The cylindrical column of air so inclosed must be set in resonance by a mobile plaque or membrane forming one end of the column, and there must be arrangements for varying either the frequency of vibration of the membrane or the length of the column, as well as for perceiving the resonance when it exists. Numerous schemes have been devised for fulfilling these requirements, but all of these involve either serious inconvenience in manipulation, or restrictions in the range of temperature available or in the material of the tube, or marked deviations from the ideal cylindrical column of gas inclosed in a rigid envelope. Thus, in Kundt's method, the tube must contain powder as well as gas; the wall must be of glass; one end must be traversed by a rod which must be free to move back and forth through the stopper, and the other by a rod which at times is in a state of violent vibration. If, instead of rendering the vibrations visible, the observer relies upon his ear to perceive them by means of the intensification of sound which they produce inside the tube, the first and second difficulties are removed; but it remains necessary to conduct the vibrations into the tube and the sound out from it. If we replace the vibrating rod by the diaphragm of a telephone receiver, the oscillations can be produced entirely inside the tube, while the introduction of a microphone as mirror permits of the sound being transmitted to the ear of the observer through the intermediary of a telephonic system. So much had been done in Germany several years ago; in particular by K pper, of Marburg, who, however, excited his receiver by means of an alternating current of constant frequency, while his microphone was shifted back and forth in the tube like the reflector in Kundt's apparatus. It is obvious that, if the to-and-fro motion of the reflector can be abolished,

the tube can be made rigid and air-tight once for all; it can be immersed completely in a temperature-bath, and kept as remote as necessary from the observer; and there are other notable simplifications in the routine, which will be described farther on. This is attained in the present work by employing a Siemens-Halske variable-frequency generator, which produces a sinusoidal alternating current of which the frequency can be easily and continuously varied over a wide range, and as easily be kept practically constant over an interval of time amply long enough to permit of its being carefully measured. In this manner, the variation in length which characterizes nearly all the methods for measuring the velocity of sound is replaced by a measurement of frequency, or essentially by a measurement of time; in absolute measurements, the length is to be determined once for all and the frequency at each observation, instead of inversely; in relative measurements, the measurement of length is entirely omitted. So far as the author knows, the only source of sound of variable frequency which has heretofore been used for experiments of this type is the siren; although at least one determination (Thiesen's) of the absolute value of the velocity of sound in air has been made with a siren playing upon a membrane closing one end of a tube, the appliance is obviously too ill-adapted to this problem to be available for general use.

The apparatus here described was first used for a determination of the speed of sound in hydrogen relatively to that in air. It then seemed desirable to make an absolute determination of the velocity in air, as well as certain auxiliary investigations upon the wall- and end-corrections. This determination was carried out at zero Centigrade, and in addition a number of values were obtained covering the range from 12° to 30° C. It was then found that while the agreement with previous values at the individual temperatures is satisfactory, the velocity as deduced from the frequency, and considered as a function of temperature, increases with temperature at a rate distinctly less than that predicted by the ordinary theory (in which γ , the ratio of specific heats, is independent of temperature). The author has not been able to find any previous paper in which the velocity is actually determined as a function of temperature over the range in question; there exist at least two investigations of the variation of velocity from 100° C. upward, and these indicate that it actually does increase less rapidly than the theory predicts; but the discrepancy between the theory (of constant γ) and the experiments mentioned (those of Stevens and Wüllner) is less marked than in the case of the evidence here presented. It is to be pointed out that these observations have all

been carried out with air in tubes, Wüllner's with a Kundt's tube, and Stevens' with a tube of a type which will later be described. Whether the discrepancies are due to peculiarities in the formation of stationary waves in tubes of various types remains to be shown, and could perhaps only be shown by means of experiments on the velocity of propagation of an acoustic signal through the free atmosphere, taken over a fairly wide range of temperature, which would be difficult, though hardly impossible; but it is believed that all the sources of experimental error in the present experiment have been eliminated, and that any discrepancy remaining over can be accounted for only by revising the form of the relationship between the frequency of a tube of this form and the velocity of propagation of acoustic waves.

Consider an infinitely long tube, traversed in opposite directions by two sinusoidal wave-trains of equal frequency ν , and hence of equal velocity v . The displacement is supposed to be uniform over any plane perpendicular to the axis, and so also is the temperature; in practice this could not be attained because of friction and thermal conduction at the walls. The effect of these boundary conditions is to retard the propagation; this is one of the important deviations from ideal conditions for which correction is required, and will be mentioned again later. If the radius of the tube is very great, the disturbance in the vicinity of the walls extends over only a negligible portion of the area of the cross-section, and may be ignored; it appears that this limiting condition may be very nearly attained, although perhaps not in the present experiment. Under these circumstances, the air-column exhibits a succession of nodes of displacement, spaced at equal intervals, these intervals being bisected by nodes of condensation; the distance between successive nodes of either system is one-half the wave-length of the tone. Imagine perfectly rigid diaphragms stretched across the tube at the positions occupied by two nodes of displacement. Since by hypothesis the air was originally motionless at the planes occupied by the diaphragms, the introduction of these rigid walls will not affect the motion of the air between them, and the portion of the air-column intercepted between the diaphragms will continue to vibrate independently of what goes on outside (so long as no energy is dissipated). We may look at this in another way; the column of air intercepted between the diaphragms is capable of an infinity of natural modes of vibration, of which the mode of n th order is characterized by the presence of $(n-1)$ nodes of displacement between the diaphragms, in addition to the nodes at each end. If we could reproduce these conditions, and measure the frequency ν_n and the internodal distance $\frac{1}{2}\lambda_n$ of the wave-system

within the tube, we could obtain the velocity of propagation of the tone of frequency ν_n inside the tube, for

$$\lambda_n \nu_n = v_n.$$

The ideal apparatus just described cannot be exactly reproduced, for it contains no provision either for producing or maintaining the tone, or for measuring the wave-length. Practical apparatus must include both mechanism for exciting the tone and mechanism for locating the nodes or at least for indicating the presence of resonance; the actual conditions thus depart from the ideal ones, and the kind and amount of departure depend upon the method.

The scheme for producing the tone which, as it appears, involves least deviation from the ideal apparatus described, depends upon a theorem which may be stated as follows: If, in our ideal tube, closed by two rigid diaphragms, one of the diaphragms vibrates along the axis with an impressed frequency ν , the node-system appears within the tube whenever ν is equal to one of the natural frequencies of the column. In other words, one of the diaphragms may be used simultaneously as a rigid wall and as the mechanism by which the vibrations inside the tube are materialized. This condition appears to be best realized in Kundt's tube, in which the vibrating diaphragm is a circular plaque mounted upon the end of a metallic rod vibrating longitudinally (but in some cases transversely, which is not so simple). In other cases a membrane of metal, mica, or rubber is stretched across the mouth of the tube, and set into vibration by the blast from a siren (Thiesen), from a pipe (Wenz *et al.*), or by electromagnetic impulses (diaphragm of a telephone receiver). The membrane must be rigidly fixed at the circumference, and hence does not conform with the theory as closely as the disk in Kundt's tube, which vibrates as a whole; but it is the only mechanism by which a tone of continuously variable frequency can be imparted to the air-column. In still other cases a tuning fork is employed, but this requires that one end of the tube should be open to the air.

The schemes for locating the nodes involve, in the main, considerable interference with the ideal conditions. In Kundt's method, powder is scattered in the tube, and is shaken up by the vibrating air-column in such a way that, when it settles down, it practically visualizes the node-system. This may be called a visual method; over against it we may set the aural methods, in which use is made of the ear. Thus in a method devised by Quincke and applied by Stevens, a slender hearing-tube is shoved back and forth along the axis of the resonance-tube; the displacement of the

small tube between two successive maxima of audible intensity is taken as the internodal distance.

Heretofore nothing has been said about the question of adjusting the impressed frequency to the length of the tube, i.e., of bringing the tube into resonance. Ordinarily this is done by displacing the rigid diaphragm. In the methods of Kundt and Stevens, the adjustment is made once for all, and the diaphragm is not moved during the observations. In other schemes, the motion of the diaphragm is made the artifice by which the internodal distance is measured. Returning to the ideal apparatus, suppose that the air-column of length l is vibrating in its n th mode, and imagine a human ear placed, say, at the first interior node of displacement counted from the rigid diaphragm, $\frac{1}{2}\lambda_n$ distant from it. Let the diaphragm be displaced outward; the audible impression should vanish instantly, or at least be much reduced. It should then reappear when the diaphragm has advanced a distance $\frac{1}{2}\lambda_n$ beyond its original position; for now the air-column is again in resonance with the impressed frequency; now, however, it is vibrating in its $(n+1)$ st mode, where before it was vibrating in its n th mode; the observer is listening at the second interior node of displacement counted from the rigid diaphragm, whereas before he was listening at the first. This is the principle in several determinations: in some, the side of the tube is pierced, and the observer listens through a rubber tube attached at the hole; in others, where the end of the tube is open and the source a tuning fork, he listens at or near the open end. Both these schemes require a notable deviation from the ideal apparatus as described.

A vast improvement was introduced by Küpper, who replaced the mirror by the diaphragm of a microphone. As is well known, the microphone, when connected in the customary way with an auxiliary receiver, transmits the maximum audible intensity to the observer's ear when the pressure-variation over the surface of its diaphragm is greatest. Now in a train of stationary waves, the pressure-variation is greatest precisely at the nodes of displacement, hence in our case at the rigid diaphragm. Hence the microphone so placed yields the maximum of audible intensity to the ear whenever the node-system appears in the tube, just as the side-tube attached at a hole opposite the position of an interior node, though in a very different manner; and it need not distort the tube in any measure. It is furthermore very advantageous for subjective reasons, as will presently appear.

Consider now a tube closed at one end by the vibrating diaphragm of a telephone receiver, at the other by the rigid diaphragm of a micro-

phone. Let the impressed frequency be ν , the length l , suppose the impressed frequency to be that of the fourth harmonic (third overtone) of the tube; then

$$2\lambda = l.$$

Let now the tube be shortened by an amount a , such that its new length is precisely such that the frequency ν is that of its second harmonic, then

$$\lambda = l - a.$$

These equations afford a means of determining ν without measuring l . It is desirable to avoid a measurement of l , for two reasons: In the first place, it is difficult to measure exactly the distance between two diaphragms inside a tube. In the second place, it is conceivable that the distance l occurring in the foregoing equations may not be exactly the distance between the diaphragms. In the case of the open tube, with one end closed and one opening into the atmosphere, it is known that the natural frequencies are integral multiples, not of the reciprocal of the distance between the ends, but of the reciprocal of this distance augmented by an additive term, the so-called "end-correction." No analogous effect has been predicted, nor so far as I know observed, in the case of closed tubes (with a single possible exception recorded by Stevens), but the possibility should be admitted for the sake of generality. If, then, we interpret l above as signifying the length inclusive of the end-correction, the equations may be solved for l and ν . In the process of solving an assumption is always tacitly made, which should for once be explicitly stated. This is the assumption that when the diaphragm-to-diaphragm distance is shortened by any amount a , the reduced length (true length plus end-correction) is diminished by the same amount—in other words, that the end-correction depends only on the nature of the ends and is independent of the length of the tube and of the order of the harmonic. On this assumption the data of the present paper are computed.

Finally, suppose that instead of shortening the tube by exactly the amount necessary to make it respond to its first overtone with the same frequency as previously made it respond in its third, we shorten it by approximately the same amount, and make the final adjustment by varying the frequency. If the amount of the shortening is very nearly correct, the small residual change in frequency will not affect the wall-correction or the other corrections in any important degree; the theory of the experiment remains the same, but the routine and details of measurement are changed immensely, and, in general, advantageously. The

equations are as follows, ν and ν' representing the resonance-frequencies of the long and short tubes, λ and λ' the corresponding wave-lengths:

$$2\lambda = l \quad \lambda' = l - a$$

$$\nu = \frac{\nu\nu'a}{2\nu' - \nu}.$$

The apparatus is built as follows: A section of brass telescope tubing, of the standard size of $2\frac{3}{8}$ inches inner and $2\frac{5}{8}$ inches outer diameter, is cut into three sections each about 40 cm. long, the severed ends being carefully smoothed so that they can be fitted evenly and tightly together. These sections will be designated as *A*, *B*, *C*. Into *A* is soldered a narrow brass ring, upon the inner face of which a screwthread is engraved; into this thread is screwed the capsule of a receiver, in such a way that the diaphragm of the receiver, framed in the brass ring, faces straight down the tube. Behind the diaphragm lies the mechanism of the receiver, from which the wires are led out through insulated plugs in an air-tight brass cap, which closes *A* at that end. Into *C* is soldered a thin brass ring, against which the diaphragm of the microphone is firmly pressed, so that the central portion of this diaphragm also faces down the tube, framed in its metal ring. Behind the microphone diaphragm lies the mechanism of the microphone, from which the wires pass out of the tube through a brass cap similar to that on *A*. Small perforations in the two brass rings allow air to pass freely from either side of either diaphragm to the other, and a side-tube attached to *C* behind the diaphragm allows air (or any other gas) to be admitted to or exhausted from the tube. *B* is left unaltered; it is the removal of *B* from between *A* and *C* which reduces the length of the tube by the quantity *a* figuring in the equations. The sections are sealed together in a rigid and air-tight manner by the use of pitch or (better) DeKhotinsky cement, applied under and around the sheaths *S'*, *S''* soldered to *B* and *C*, respectively.

The microphone or transmitter is one of the standard types of the Western Electric Company, Number 291-W, selected because its diaphragm is very nearly flat, without projections on the outer side. This was connected through one or two dry cells to the primary of a standard telephone induction-coil also from the Western Electric, the secondary of which was connected to an ordinary receiver, hereafter distinguished as the "auxiliary receiver."

The receiver which produces the tone is by the Wireless Specialty Company, for use in radiotelegraphy; it showed itself greatly superior to the receivers used in ordinary telephony, as well as to a receiver

made by another wireless supply company, in respect of the exceedingly pure, musical tone which it emitted when energized by the alternating current employed.

The alternating current was supplied from a Siemens-Halske variable-frequency generator. This consists essentially of a toothed wheel of laminated iron, of which the teeth pass very closely above the two teeth of a U-shaped yoke, also in laminated iron, around which are wound two coils. A direct current of about 3 amperes is passed through one of the coils; the magnetic flux excited by this current in the yoke completes its circuit through the wheel, and has a high value when teeth of the wheel are over the teeth of the yoke, and a low value when troughs of the wheel are above the teeth of the yoke. The wheel is mounted upon the axle of a motor, so that when the motor is rotating m times in a second, and the wheel has p teeth, an approximately sinusoidal current of frequency mp flows through the second coil; this is also the frequency of the primary receiver, to which the second coil is connected. The determination of the frequency ν , therefore, reduces to the determination of the speed of the motor. The motor is a Holtzer-Cabot type, used as a shunt-wound motor, and operated with the storage-battery of the laboratory at a speed of some 1,800 r.p.m. The original German motor belonging to the apparatus had to be rejected because of the difficulty of keeping the speed constant. The wheel employed had thirty teeth, so that the frequency attained with a speed of 1,800 r.p.m. was about 900.

The air is drawn from outdoors through a tube passing through the window frame, and traverses washing-bottles of sulphuric acid and potassium hydroxide solution, followed by tubes of soda-lime and phosphorus pentoxide. As the weather was cold and dry during the period of most of the observations, the original vapor-pressure would have been exceedingly small, and the P_2O_5 tube never showed more than a trace of alteration.

The distance a is obtained by engraving a longitudinal scratch upon the tube, and two transverse scratches, one on A and one on C , intersecting the long scratch; the distance between the two intersections is measured when the section B intervenes, and again when it is removed, and the difference between the two distances is taken as d ; the long scratch incidentally serves to indicate the original alignment when the central section is inserted or extracted. The measurement is made in the conventional manner, the movable cross-hairs of rigidly stationary micrometer-microscopes being adjusted upon the real images of the scratches at the intersections; the tube is then removed and replaced by the standard scale, interpolation between successive scratches on this

latter being made by means of the micrometer. The scale is by the Société G  n  voise, and is engraved upon a thin, silver lamina incased in a massive brass rod; since the tube is also of brass, the measurements, at whatever temperature made, are correct at the temperature for which the scale is guaranteed, this being zero Centigrade. The scale is guaranteed accurate to "one or two hundredths of a millimetre"; since the distances, of which a is the difference, are, respectively, about 63 and about 25 cm., the total uncertainty from this cause should not exceed .03 mm., which is about one part in fifteen thousand of a . It is easy to bring the observational error well within this limit; I append the following sample:

$$\begin{array}{lcl} \text{Long tube} & \left\{ \begin{array}{l} 629.242 \\ 629.226 \\ 629.225 \\ 629.224 \end{array} \right. & \begin{array}{l} \text{Short tube} \left\{ \begin{array}{l} 249.526 \\ 249.531 \\ 249.535 \end{array} \right. \quad a = 379.70. \end{array} \end{array}$$

A higher degree of accuracy could be attained without difficulty, but seems nugatory, especially as it is impossible to be sure that the diaphragm of the receiver retains the same position, to within a hundredth of a millimeter, under the influence of its own electromagnets, or of the shocks inevitable when the length of the tube is altered. In plotting the frequency of the tube against the temperature, the observed frequencies are reduced to the value they would possess if the tube retained its length at zero; for the purpose of this reduction, the coefficient of thermal expansion of the tube is taken as 0.000019.

The temperature is taken as that of the water-bath in which the tube is immersed, and is measured by means of a thermometer of which the scale is divided into tenths of a degree, and can easily be read to hundredths. Uniformity of temperature is attained by stirring the water repeatedly and violently, at least once in five minutes during the readings; this will be brought up later.

The determination of the resonance-frequency, which is the peculiar and cardinal measurement of the whole process, is carried out by two observers working conjointly; one has the auxiliary receiver constantly at his ear and controls the speed of the motor by means of a slide-wire rheostat in series with the motor armature, the other measures the speed of the motor while the first holds it steady at what he deems to be the value corresponding to maximum intensity of sound in the auxiliary receiver. The assistant carries out the actual measurement by noting down the time in which a speed-counter, attached to the axle of the motor, describes eight revolutions. The time-measuring instrument is a Peyer-

Favarget chronoscope, which is an apparatus which records, by means of a pointer revolving in front of a dial, the interval between the closing and the opening of an electrical circuit. This make and break may be performed manually, but is much better done by an ingenious mechanical device for which I am indebted to a former assistant, Mr. W. R. Baldwin.

The terminals of the chronoscope circuit are led, one to a mercury cup and the other to an iron lever, one arm of which ends in a prong with a downward-pointing platinum tip; under each arm is a small solenoid; when the solenoid under the arm with the platinum tip is traversed by a momentary current, the arm is pulled down, and the tip plunges into the mercury, closing the circuit and starting the chronoscope; it so remains until a current passes through the other solenoid, when the opposite arm is pulled down and the platinum point pulled out of the mercury, stopping the chronoscope. To the dial of the speed-counter is soldered an iron prong, bending vertically downward so as to graze the surface of another mercury cup once in each revolution, thus closing a second circuit which either passes through one solenoid, or through the other solenoid, or is interrupted, according to the position of a two-way switch which the assistant controls. He first closes the switch in such a way that, when the prong on the dial enters the mercury, the current passes through the solenoid first mentioned, and starts the chronoscope; he then opens the switch and allows the chronoscope to continue in motion during eight revolutions of the speed-counter; just before the prong enters the mercury for the ninth time, he throws the switch over to the other side so that the first contact sends the current through the second solenoid, and stops the chronoscope.

The difference between the initial and the final readings indicated by the pointer on the dial give the time elapsed between the opening and the closing of the chronoscope circuit, in terms of scale-divisions of the dial, of which the least is about equivalent to one five-hundredth of a second. In order to reduce to standard seconds, the prong projecting from the dial of the speed-counter is replaced by the hammer of a relay, which is governed by the escapement of the laboratory clock, and clicks once in two seconds, or twenty-nine times in a minute, the thirtieth click being omitted so as to provide a convenient zero-point; the assistant, by throwing the switch previously mentioned to one side between the third and fourth clicks (counting from the omitted click) and to the other side between the nineteenth and twentieth, causes the chronoscope to register the equivalent of thirty-two seconds of the clock in chronoscope units. As this operation is completely controlled by mechanism, it is particularly

apt for estimating the constancy of speed of the chronoscope. It is found that four or five readings may often be taken in succession, agreeing within one unit of the chronoscope; as there is only one point in each scale-division of the latter at which the pointer can stop, this is as good a constancy as could possibly be demanded. On the other hand, a slight trend of the chronoscope calibration-readings may generally be observed when the operation is prolonged over fifteen minutes or more; the apparatus seems to run more slowly by three to five parts in fifteen thousand, after a set of readings half-an-hour long, than before. For this reason, each set of measurements upon the speed of the motor is preceded by a certain number (usually ten) of calibration-readings upon the length of the interval from the fourth to the twentieth click, and is followed by an equal number; the mean of the two means so obtained is employed as the equivalent of thirty-two seconds for the set of speed-readings, after the application of three corrections to be described.

The first correction is for what may be called the "personal equation" of the time-measuring apparatus. The closing of the auxiliary circuit through the first solenoid will precede the closing of the chronoscope circuit by an interval which will doubtless be finite and perhaps appreciable; similarly, the closing of the auxiliary circuit through the second solenoid will precede the opening of the chronoscope circuit by an interval which may be appreciable, and it is not safely to be presumed that these intervals will be equal, i.e., that the interval between the closing and the opening of the chronoscope circuit will be equal to that between the closing of the auxiliary circuit through the first and that through the second solenoid. To evaluate the difference, let T be the time recorded by the chronoscope between the fourth and the twentieth click; T' , that between the fourth and twelfth; T'' , that between the twelfth and twentieth; let $T+x$, $T'+x$, $T''+x$ represent the true time between the corresponding clicks; then

$$T+x = T'+x + T''+x.$$

The quantity x is to be added to all chronoscope readings. It is determined in practice by taking a series of calibration-readings during thirty consecutive minutes of the clock, ten being on the interval from the fourth to the twentieth click, ten on that from the fourth to the twelfth, and ten on that from the twelfth to the twentieth. A typical set of readings is given in Table I. Four sets of readings on four successive days gave for x the values 0.116, 0.116, 0.117, 0.110. Some months previously the observed values had averaged 0.083; but since the correction is added both to the speed-readings and to the calibration-readings, i.e., to both

numerator and denominator of a quotient to which the frequency is proportional, the uncertainty thus introduced into the final result is quite negligible (of the order of one part in twenty thousand).

TABLE I

PREVIOUS CALIBRATION	A. TYPICAL DETERMINATION OF FREQUENCY			B. PERSONAL EQUATION		
	Readings on Speed; 8 Revolutions Speed-Counter In		Subsequent Calibration	I	II	III
160.69	131.96	20°00	131.93	160.68	80.24	80.30
160.70	131.99		131.99	160.67	80.65	80.29
160.67	132.02		131.98	160.67	80.64	80.30
160.68	132.03		131.92	160.66	80.63	80.29
160.68	132.02		131.95	160.68	80.63	80.21
160.68	132.09	20.00	19°95	160.66	80.62	80.29
160.67	131.93			160.70	80.62	80.29
160.66	131.99		131.97	160.69	80.62	80.29
160.69	131.95		131.96	160.67	80.64	80.28
160.70	132.02		131.97	160.67	80.60	80.28
160.682	132.02		132.04	160.675	160.628	80.225
			19.92			
	131.97		131.982			
	131.96	19.97				

Eight revolutions of speed-counter=800 revolutions of motor=24,000 pulsations of emitting diaphragm take place in 131.98 chronoscope units.

160.68 units of chronoscope=32.007 standard seconds.

Hence, number of pulsations of emitting diaphragm in one second=frequency of resonance of (short) tube (allowing for personal equation $x=0.12$).

$$\nu = \frac{160.68 + 0.12}{131.98 + 0.12} \frac{24000}{32.008} = 912.71 \text{ at } 19^{\circ}97.$$

Personal equation: column I contains values of T ; II those of T' ; III those of T'' .

$$T: 160.628 \quad T'=80.225 \quad T''=80.287. \\ x=+0.116$$

It will be observed in Table I that the interval b from the fourth to the twelfth click is not equal to the interval c from the twelfth to the twentieth, although each should be exactly sixteen seconds. This disparity is due to irregularities in the clock and in the mechanism for transmitting time-signals from the clock, which make it a priori uncertain whether either is sixteen seconds, i.e., $16/86400$ of the solar day, exactly. The same objections apply to the interval from the fourth to the twentieth click, but not to any two-minute interval, i.e., to the interval from any click to the sixtieth click following, which may be taken as $1/720$ of the

twenty-four hours of the clock, exactly. The ratio of the interval from the fourth to the twentieth click to the two-minute interval is therefore determined by special sets of readings, taken over a half-hour period on the two intervals alternately.

Finally, the rate of the clock is determined by comparison with the standard time obtained from Arlington by wireless; a number of comparisons, made at various times over a period of several months, give values varying between $1\frac{1}{4}$ and $2\frac{1}{2}$ seconds *per diem*, positive; the correction is of the order of one part in forty thousand, which is negligible. The interval from any click to the sixtieth click following is therefore 120 standard seconds, practically; the interval from the fourth to the twentieth is $160.78/602.77$ of this, and hence equal to 32.008 standard seconds.

The speed-counter revolves once for each 100 revolutions of the motor, hence for each 3,000 alternations of the current in the primary receiver; that this is so is verified by affixing a marker to one of the thirty teeth of the toothed wheel, and observing whether after twenty complete revolutions of the speed-counter the marked tooth returns to the same point of the circle as before; it is observed that the coincidence of the initial with the final orientation is frequently perfect, so far as the eye can tell, and that in the most unfavorable cases the deviation is less than one-sixth of a complete turn, while the wheel has actually described 20×100 or 2,000 turns; and these deviations are as likely to be positive as negative, which shows that in all probability they come about not because the factor of the speed-counter differs from 100, but because of such things as slight differences in the mercury level, the point at which the prong makes electrical contact, etc., affecting the initial and the final contact equally, and balancing out in the average.

The task of the other observer, who keeps the speed of the motor at the proper value while the assistant operates the chronoscope and records the speed, unfortunately cannot be delegated to a machine. It is here that subjective errors may appear. Generally speaking, the resonances of such a tube as is here used are surprisingly sharp and clear, far more so than those produced in the other aural methods; they may be likened to sharp spectrum lines, while those usually obtained are more like hazy bands. Thus, many observers state that they were in doubt as to whether to make their settlements upon the maxima or the minima of intensity; no such doubt or choice could exist in this case, for the intensification of sound due to resonance is entirely restricted to a range of frequency less than one-hundredth of the interval between two resonances of different order.

The function of the primary observer is, then, to set and hold the speed of the motor as nearly as possible at the rate which produces the maximum resonance in the tube, and the maximum sound in the receiver which he holds at his ear. In order to do this, he has at his hand a slide-wire rheostat connected in series with the armature of the motor. Since the speed of the motor will not in any case long remain steady, it appears to lead to the most uniform results to vary this speed very rapidly back and forth across the value which gives maximum resonance. Sufficient variation is obtained by moving the slider back and forth over a millimeter or two with the thumb and forefinger, with from twenty to thirty to-and-fro motions in the thirty-second period over which a single reading by the assistant extends. This maneuver soon becomes instinctive, indeed some care is necessary to prevent the formation of a habit in which, e.g., a systematic error might be produced by a predilection for keeping the speed above the resonance-speed for a larger part of the time than below it. Emphasis should be laid on the ease of this manipulation; for all the aural methods require that the observer should be doing something with his hands while he is listening to the resonance, and this appears to be the most facile. It is certainly easier, for example, to move the slider of the rheostat than it is to draw a tightly fitting piston back and forth within the tube. Some of the early experimenters even had to strike tuning forks continually during the process. Nor is this an unimportant matter, since the accuracy of the readings is seriously marred by anything which requires the dissipation of attention from the quality of the sound itself. Again, in all the previous methods the observer must hold his ear very close to the resounding tube and to the source as well; this diminishes the relative intensity of the resonance and in addition admits the possibility of misleading interferences, as has been recognized by many experimenters. In the present method, the observer may be as remote as he desires from the tube and from the motor.

A full set of readings ordinarily consists of twenty-five or thirty single readings, preceded by ten and followed by ten calibration readings. The full set is divided into five sets of six each, between each pair of which, as well as before and after the full set, the water in the trough is thoroughly stirred and the temperature read. The sets of six are split into three parts of two readings each, between which the microphone is disconnected for a few seconds; this is done to minimize any possibility of subjective error due to a conceivable tendency of the ear to abide by its first decision as to the point of maximum intensity.

The data are now submitted, beginning with those relating to zero Centigrade. During the course of observations the tube was once lengthened and once shortened. With the tube at its initial length, the section *B* being inserted, two values of the frequency were obtained, of which the squares are 76,249, 76,279. The tube was then shortened, the amount of the shortening, as measured by the diminution in the distance between the two transverse scratches before mentioned, being 379.774 mm. With the short tube, three values of the frequency were obtained, their squares being 76,533, 76,499, 76,493. The tube was then lengthened again, and six values of ν determined, their squares being 76,245, 76,236, 76,251, 76,284, 76,207, 76,214. Unfortunately it was not possible to make further tests upon the short tube, as the application of heat in an attempt to determine the frequency at 100° loosened the diaphragms and impaired the rigidity of the tube.

In order to compare the initial with the final values obtained with the long tube, it is necessary to consider that, during the last set of readings, the distance between the scratches, and hence presumably that between the diaphragms, was .07 mm. (out of 76 cm.) greater than during the first set; hence if we employ for a the value obtained when the tube was lengthened, viz., $a = 379.70$ mm., the two frequencies obtained at first must be increased by one part in ten thousand. Hence we have:

Long tube: ν^2 :	76,263	Short tube: ν'^2 :	76,533	Mean ν^2 :	76,256
	76,293		76,499	Mean ν :	
	76,245		76,493	Mean ν'^2 :	76,508
	76,236			Mean ν' :	
	76,251				$a_0 = 379.70$
	76,284				
	76,207				
	76,214				

whence

$$\nu = 331.03.$$

The probable errors of the values of ν^2 and ν'^2 are of the order of one part in eight thousand, which is halved for the frequencies themselves. The peculiar way in which ν and ν' are combined in the expression for ν makes possible a combination of errors (if ν is too great and ν' too small, or vice versa) which would make the uncertainty in ν twice that in the frequencies individually. Finally, the uncertainty in a due to the possible irregularities of the scale is of such magnitude as to render the total probable error equal about to one unit in the first decimal place, so that

$$\nu_0 = 331.03 \pm .10.$$

In addition to the determination of ν and ν' at zero, a number of observations were made at various temperatures between 12° and 30° with the object of plotting the relationship between velocity and temperature. The results, which are tabulated in Table II (p. 22), the square of the frequency being given for each temperature, conform in a very satisfactory manner with the prediction of the elementary theory, in so far as the square of the frequency is, as far as the accuracy of the observations permits of judging, a strictly-linear function of the temperature over the range from 0° to 30° . The slope of the line is, however, less than that predicted by the ordinary theory, in which γ , the ratio of the specific heats, is assumed to be constant. In other words: whereas, if γ does not depend upon the temperature, the line which represents ν^2 as a function of T should, if prolonged backward, intersect the axis of temperature at the absolute zero (as computed from the constants of air, which for the present purpose is sufficiently nearly an ideal gas in this range to be so considered)—the actual line intersects the axis of abscissae at (about) -295° . In the table, the frequencies given are not the actual frequencies at the corresponding temperatures, but the actual frequencies, reduced, so as to apply to a tube having, at all temperatures, the same length as the actual tube has at zero (the coefficient of thermal expansion of brass being taken as .000019). By theory, we have for (say) the short tube:

$$(\nu')^2 = \frac{v^2}{\lambda^2} = \frac{v^2}{l_0^2} = \frac{pV\gamma}{l_0^2} = \frac{p_0 V_0 \gamma}{l_0^2} (1 + \alpha T) = \frac{RT_0 \gamma}{l_0^2} (1 + \alpha T) = (\nu'_0)^2 (1 + \alpha T),$$

where l_0 is the length of the tube (plus end-correction) at zero, T_0 the absolute temperature of the Centigrade zero, V the reciprocal of the density (which is approximately the same in all the readings), and α the so-called coefficient of expansion at constant volume; the latter equations involve the assumption that γ does not depend on temperature. From these equations it follows that the slope of our line should stand in a definite ratio to the value of its intercept on the $(\nu')^2$ axis at zero Centigrade; viz.:

$$\text{Theor. slope} = (\nu'_0)^2 (\alpha) = 765080 \times .003665 = 2804,$$

whereas we have

$$\text{Actual slope} = 2616 \text{ approximately.}$$

The observed rate of increase with rising temperature of the frequencies of the tubes (for the slope of the line for the long tube is indistinguishable from that for the short one) is therefore markedly less than

the supposed rate of increase of the velocity of sound. It should be possible to control this observation by comparison with the results of other observers using other methods; but I have not been able to find any recorded investigations in which the individual observations are both accurate enough and spread over a sufficiently wide range to make it possible to plot a comparative curve over the same temperature-interval. The observations of Stevens and Wüllner at 100° C. indicate that the velocity of sound does actually increase less rapidly with temperature than is to be expected if γ is constant, but the deviation of their results from the theory is less marked than that of the results here presented. Thus, if we assume that the effect is entirely due to a diminution in γ with increasing temperature (as did Stevens and Wüllner), it is necessary to suppose that γ diminishes by over 1 per cent as the temperature rises from 0° to 100° , while Wüllner and Stevens find a diminution of only a small fraction of 1 per cent. The amount of this divergence practically excludes trivial errors from consideration.

It remains to consider the influence of the walls. According to all theory and experiment, this should be a retardation, diminishing in proportional amount as the frequency rises. As the rise in temperature entrains, with this method, a rise in frequency, the line should be tilted upward, or in the opposite sense from the one in which it is actually deflected. However, the present knowledge about the dependence of velocity on diameter in tubes is limited only to room-temperature and almost entirely to glass tubes, and it is possible that some direct relationship may exist between the temperature of the wall and the magnitude of the retardation. Such a hypothesis might reconcile the present evidence with that of Stevens and Wüllner, for the former experiment was performed with a porcelain, the second with a glass tube; it is quite certain that at room-temperature the correction is much greater for a metal tube than for a glass or porcelain tube, and it would seem probable that if the retardation does increase with temperature, the increase would be markedly greater with metal than with glass. It might also abolish the difficulty in the kinetic theory of diatomic gases introduced by the experiments of Stevens, which indicate that the value of γ sinks below 1.4 as the temperature rises beyond 100° .

The wall-correction is also required to reduce the observed velocity to that in free space; but for metal tubes (and indeed for glass tubes as well) its amount even at room-temperature is so uncertain that little weight can be attached to the result. Too much reliance is sometimes placed in Kirchhoff's formula, which does not appear adequately to rep-

resent the facts except in so far as it predicts that the influence of the walls is to produce a retardation, which diminishes, with increasing frequency and increasing radius of the tube. A considerable literature of this subject already exists, the favorite type of experiment being that in which several glass tubes of different diameter and several sources of different frequency are used, and the investigator seeks the value of Kirchhoff's constant which, when applied to the various observed speeds, reduces them to most nearly the same asymptotic speed. A study of the literature must lead to the conclusion that no one choice of Kirchhoff's constant, in particular not the value which Kirchhoff's own theory assigns to it, will make his formula fit all the cases already investigated in the study of glass tubes, while the whole question of the difference between metal tubes and glass tubes in this regard is left unsettled. I append, however, the values to which the foregoing value of v_0 , viz., 331.03, is raised if we apply Kirchhoff's formula with (a) his value of the constant, .00588, and (b) Kayser's empirical value, .0235, adopted by Violle and Vautier,

v_0 in a brass tube of radius 31 mm.	331.03
v for free space (Kirchhoff's formula)	331.33
v for free space (Kayser's constant)	332.26

In conclusion, some values may be given for comparison; it is to be noted that comparisons should, strictly, be made at the temperature at which the actual readings are taken, and this is in many cases impossible, because the investigators in question have not recorded the temperatures at which they actually worked, publishing only the values reduced to 0 C.; among these are Kayser, Blaikley, and in part, Regnault. Wüllner and Thiesen worked at 0° C.; the former, with Kundt's method, a glass tube of 3 cm. diameter, and an impressed frequency $\nu=2539$, obtains 331.90 without any correction for the walls; the latter, working with the higher overtones of a tube almost exactly like mine in size and material, excited by a siren blowing against one end, but using a scheme for perceiving the resonance much less delicate than mine, obtains 331.92 from the higher overtones with Kirchhoff's formula. Stevens gives three sets of observations at temperature between 19° and 21°, his data as recorded being already reduced to 0°; reducing them backward with his value of the coefficient of expansion of air, and averaging them at his mean temperature of 20°, I find that he obtains 343.25 by the use of Kirchhoff's formula with .0074 as constant; my value at the same temperature with the same correction comes out 343.0. Hebb has three very concordant readings at 17°5, giving 341.63 in free space; I obtain

341.1 *uncorrected* at the same temperature; at 22°5 he has 344.8, I obtain 343.9 *uncorrected*.

In summary, the following conclusions are submitted:

1. It is possible to determine, conveniently and with a high order of accuracy, the resonance-frequency of a column of gas of constant length comprised between the diaphragm of a telephone receiver and the diaphragm of a microphone, by exciting the receiver with an alternating current of variable frequency from a variable-frequency generator and employing the microphone to detect the existence of a state of resonance.

2. By making such a determination with two tubes differing only in length, and by a measurable amount in length, it is possible to determine the velocity of sound in the gas, the determination reposing only upon the assumption that the end-correction of the gas-column is independent of its length.

3. The resonance-frequency of such a tube filled with air increases less rapidly with temperature than is to be expected from the classical theory, the deviation being about 3.5 per cent of the theoretical rate of increase; it is the same for a tube 76 cm. long as for a tube 38 cm. long, so that the velocity of sound in air in a tube of the actual material and dimensions—brass and 62 millimeters in diameter—must increase less rapidly than the theory predicts (i.e., less rapidly than the square root of the absolute temperature) by the same percentage.

4. The velocity of sound in a brass tube of 62 millimeters diameter, at zero Centigrade, is (by direct measurement at zero):

$$v_0 = 331.03 \text{ meters per second,}$$

with a probable error of 0.10.

I wish gratefully to record the co-operation of Mr. Merwin J. Kelly, on whom devolved the tedious task of carrying out the speed-readings and most of the calibration-readings of the chronoscope.

(Written in May, 1917.)

TABLE II
DATA USED IN PLOTTING v^2 AS FUNCTION OF T

Long Tube....	12.34	135.143	160.64	79,464
	15.96	134.242	160.576	80,483
	20.09	133.276	160.491	81,568
	25.96	132.056	160.490	83,104
	23.96	132.429	160.46	82,600
	21.95	132.920	160.54	82,068
	30.20	131.286	160.59	84,196
	28.04	131.588	160.53	83,742
	13.60	134.573	160.49	80,025
Short Tube...	27.25	131.856	160.695	83,578
	29.96	131.280	160.676	84,301
	31.30	130.973	160.669	84,694
	25.71	132.075	160.652	83,252
	21.04	133.059	160.659	82,053
	19.97	133.274	160.669	81,760
	23.92	132.456	160.643	82,755
	17.87	133.693	160.674	81,248
	21.61	132.907	160.632	82,179
	24.44	132.332	160.612	82,884
	25.62	132.129	160.621	83,187

The readings are given in chronological order. The first column gives the temperature; the second the time of 8 revolutions of the speed counter in units of the chronoscope; the third the time of 16 seconds of the clock in units of the chronoscope; the fourth the square of the frequency, corrected for personal equation, for the ratio of the 16-second to the 120-second interval, and for the expansion of the tube with increasing temperature.

REFERENCES

- D. J. Blaikley, *Phil. Mag.*, **18** (1884), 328-44.
 K. K. Darrow, *Phys. Rev.* (2), **7** (1916), 413-14.
 T. C. Hebb, *Phys. Rev.*, **20** (1905), 89-99.
 H. Kayser, *Wied. Ann.*, **2** (1877), 218-41; **6** (1878), 465-85.
 W. Küpper, *Ann. d. Phys.*, **43** (1914), 905-28.
 H. V. Regnault, *Memoires de l'Academie*, **37** (1868).
 E. H. Stevens, *Ann. d. Phys.*, **7** (1902), 285-320.
 M. Thiesen, *Ann. d. Phys.*, **25** (1908), 506-20.
 J. Violle and T. Vautier, *Ann. Chim. Phys.* (6), **19** (1890), 306-45.
 W. Wenz, *Ann. d. Phys.*, **33** (1910), 951-70.
 A. Wüllner, *Wied. Ann.*, **4** (1878), 321-41.

